Longitudinal Data Analysis I

PSYC 575

October 3, 2020 (updated: 10 October 2021)

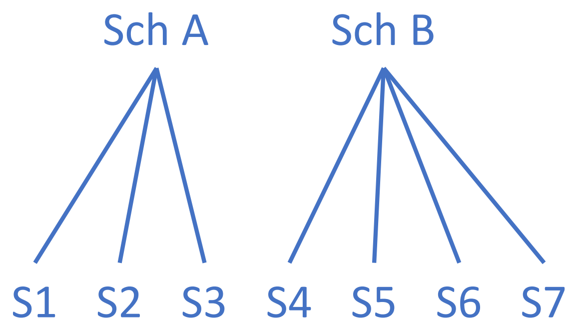
Learning Objectives

- Describe the similarities and differences between **longitudinal data** and cross-sectional clustered data
- Perform some basic attrition analyses
- Specify and run **growth curve analysis**
- Analyze models with **time-invariant covariates** (i.e., lv-2 predictors) and interpret the results

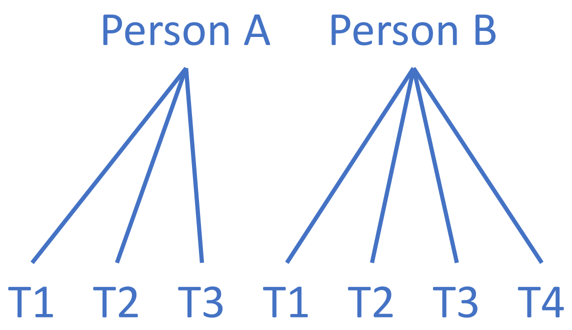
Longitudinal Data and Models

Data Structure

- Students in Schools



- Repeated measures within individuals



Types of Longitudinal Data

- Panel data
  - Everyone measured at the same time (e.g., every two years)
- Intensive longitudinal data
  - Each person measured at many time points
  - E.g., daily diary, ecological momentary assessment (EMA)

Two Different Goals of Longitudinal Models

- Trend
  - Growth modeling
  - Stable pattern
  - E.g., trajectory of cognitive functioning over five years
- Fluctuations
  - Clear trend not expected
  - E.g., fluctuation of mood in a day

Example

Children's Development in Reading Skill and Antisocial Behavior

- 405 children within first two years entering elementary school
- 2-year intervals between 1986 and 1992
- Age = 6 to 8 years at baseline

Same Multilevel Structure

- At first, it may not be obvious looking at the data (in wide format)

id <dbl>	anti1 <dbl>	anti2 <dbl>	anti3 <dbl>	anti4 <dbl>	read1 <dbl>	read2 <dbl>	read3 <dbl>	read4 <dbl>
22	1	2	NA	NA	2.1	3.9	NA	NA
34	3	6	4	5	2.1	2.9	4.5	4.5
58	0	2	0	1	2.3	4.5	4.2	4.6
122	0	3	1	1	3.7	8.0	NA	NA
125	1	1	2	1	2.3	3.8	4.3	6.2
133	3	4	3	5	1.8	2.6	4.1	4.0
163	5	4	5	5	3.5	4.8	5.8	7.5
190	0	NA	NA	0	2.9	6.1	NA	NA
227	0	0	2	1	1.8	3.8	4.0	NA
248	1	2	2	0	3.5	5.7	7.0	6.9
	T1	T2	T3	T4	T1	T2	T3	T4

Restructuring!

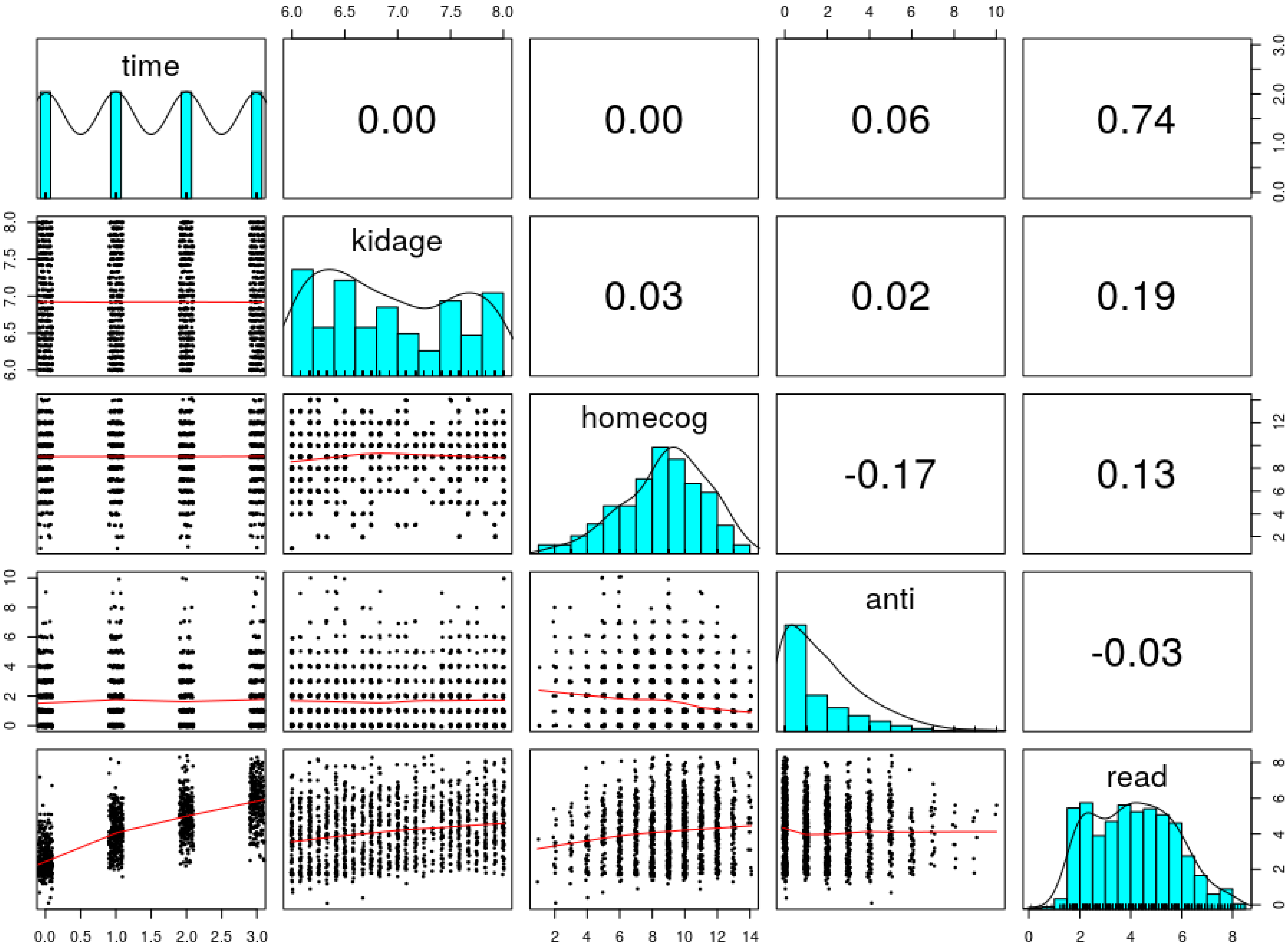
- Long format

"Cluster" 22

id	anti	read	time
<dbl>	<dbl>	<dbl>	<dbl>
22	1	2.1	1
22	2	3.9	2
22	NA	NA	3
22	NA	NA	4
34	3	2.1	1
34	6	2.9	2
34	4	4.5	3
34	5	4.5	4
58	0	2.3	1
58	2	4.5	2

id	anti	read	time
<dbl>	<dbl>	<dbl>	<dbl>
58	0	4.2	3
58	1	4.6	4
122	0	3.7	1
122	3	8.0	2
122	1	NA	3
122	1	NA	4
125	1	2.3	1
125	1	3.8	2
125	2	4.3	3
125	1	6.2	4

id	anti	read	time
<dbl>	<dbl>	<dbl>	<dbl>
133	3	1.8	1
133	4	2.6	2
133	3	4.1	3
133	5	4.0	4
163	5	3.5	1
163	4	4.8	2
163	5	5.8	3
163	5	7.5	4
190	0	2.9	1
190	NA	6.1	2

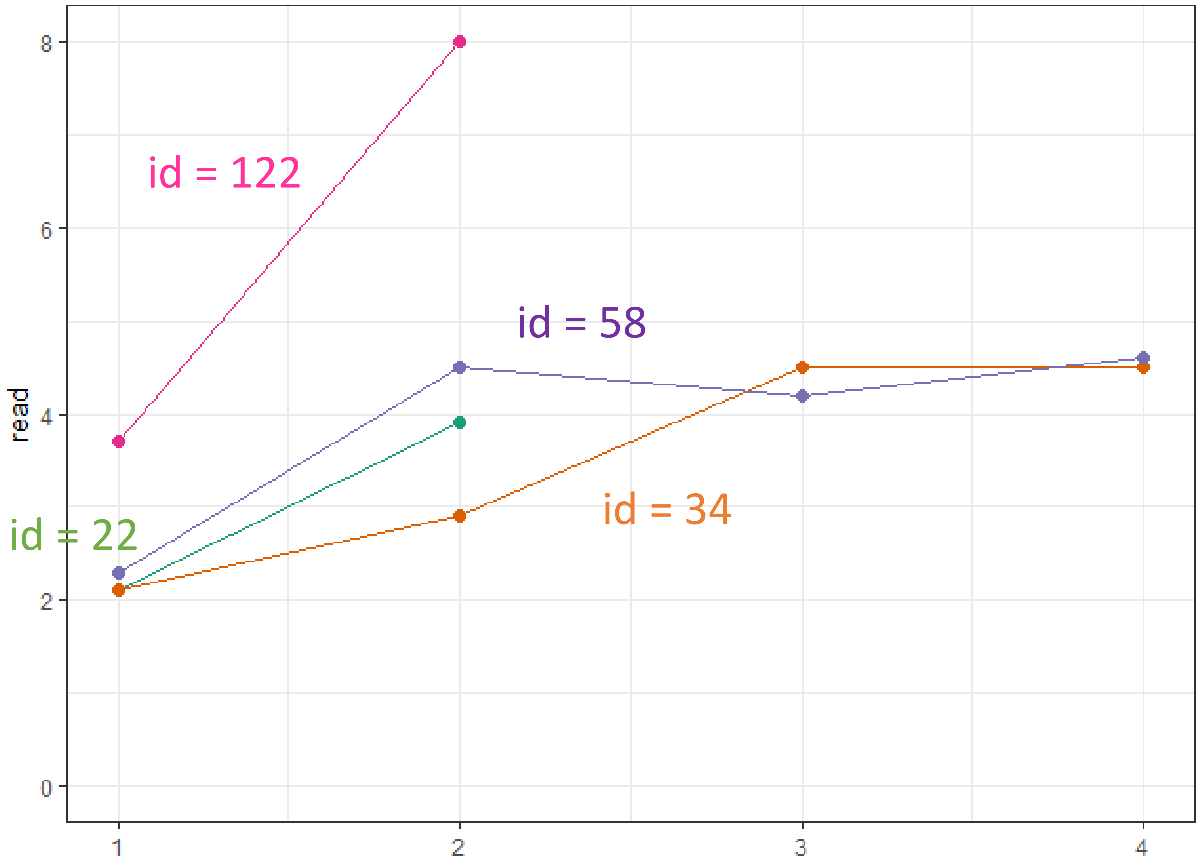

time
kidage
homecog
anti
read
0.00
0.00
0.06
0.74
0.03
0.02
0.19
-0.17
0.13
-0.03

Attrition Analysis

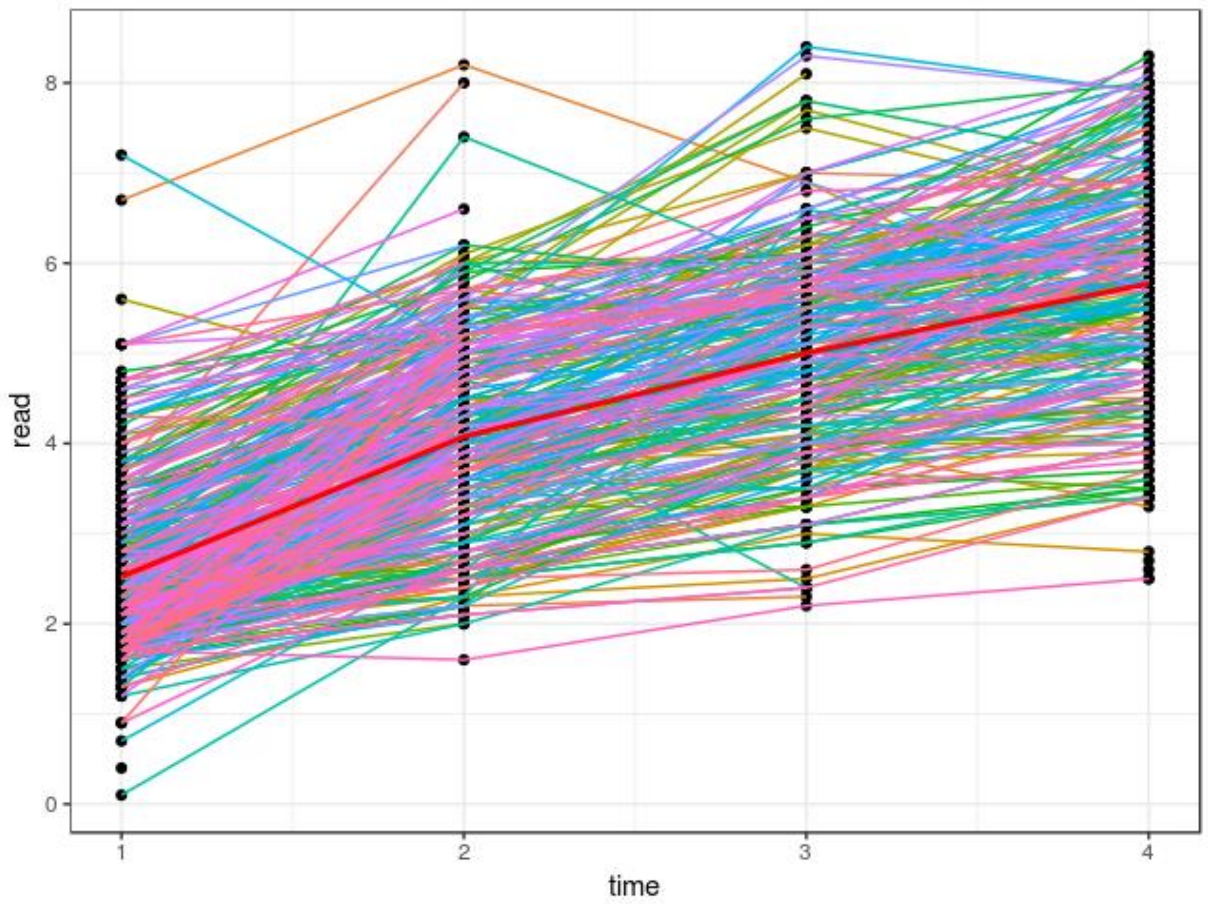
- Whether those who dropped out differ in important characteristics than those who stayed
- Design: Collect information on predictors of attrition, and perceived likelihood of dropping out
- Limited generalizability
- Missing data handling techniques
  - E.g., Multiple imputation, pattern mixture models

	complete		incomplete	
	Mean	SD	Mean	SD
anti1	1.49	1.54	1.89	1.78
read1	2.50	0.88	2.55	0.99
kidgen	0.52	0.50	0.48	0.50
momage	25.61	1.85	25.42	1.92
kidage	6.90	0.62	6.97	0.66
homecog	9.09	2.46	8.63	2.70
homeemo	9.35	2.23	9.01	2.41

Visualizing Some “Clusters”



Spaghetti Plot



Growth Curve Modeling

MLM for Longitudinal Data

	Student *i* in School *j*	Repeated measures at time *t* for Person *i*
Lv-1 model	$MATH_{ij} = \beta_{0j} + \beta_{1j}\, SES_{ij} + e_{ij}$	$READ_{ti} = \beta_{0i} + \beta_{1i}\, TIME_{ti} + e_{ti}$
Lv-2 model	$\beta_{0j} = \gamma_{00} + u_{0j}$ $\beta_{1j} = \gamma_{10} + u_{1j}$	$\beta_{0i} = \gamma_{00} + u_{0i}$ $\beta_{1i} = \gamma_{10} + u_{1i}$
Random effects	$\mathrm{Var}\begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} = \begin{bmatrix} \tau_0^2 & \tau_{01} \\ \tau_{01} & \tau_1^2 \end{bmatrix}$ $\mathrm{Var}(e_{ij}) = \sigma^2$ τ_0^2, τ_1^2 = intercept & slope variance *between* schools σ^2 = *within*-school variation (across students)	$\mathrm{Var}\begin{pmatrix} u_{0i} \\ u_{1i} \end{pmatrix} = \begin{bmatrix} \tau_0^2 & \tau_{01} \\ \tau_{01} & \tau_1^2 \end{bmatrix}$ $\mathrm{Var}(e_{ti}) = \sigma^2$ τ_0^2, τ_1^2 = intercept & slope variance *between* persons σ^2 = *within*-person variation (across time)

Random Intercept Model (with **glmmTMB**)

```
> m00 <- glmmTMB(read ~ (1 | id), data = curran_long, REML = TRUE)
> summary(m00)

Random effects:

Conditional model:
 Groups   Name        Variance Std.Dev.
 id       (Intercept) 0.3005   0.5482
 Residual             2.3903   1.5461
```

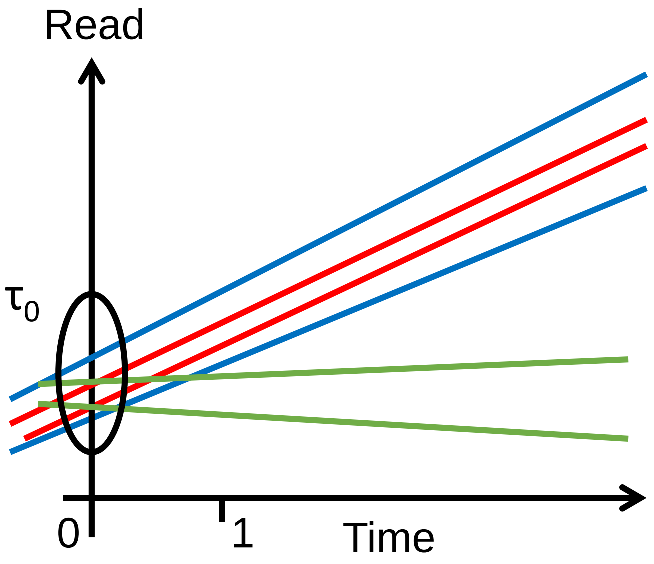
- Estimated ICC = 0.11

Linear Growth Model

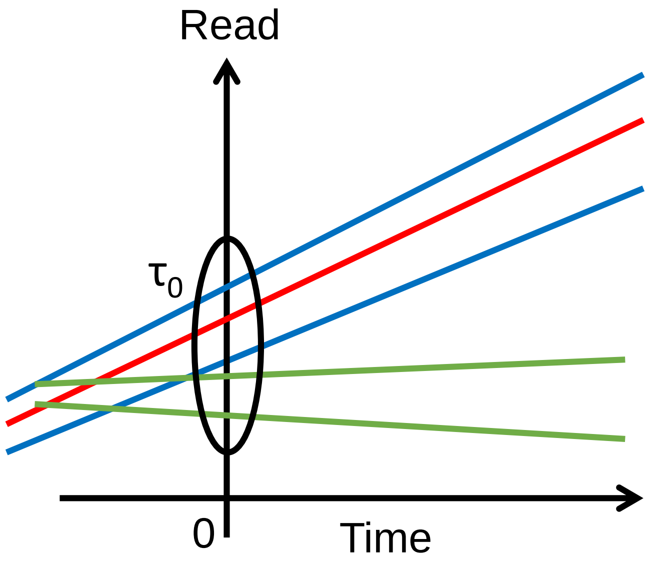
- Here time is treated as a continuous variable
  - Can handle varying occasions
  - Assume time is an *interval* variable
- Fit a linear regression line between time and outcome for each "cluster" (individual)

(Grand) Centering of Time

- Time = 1, 2, 3, 4



- Time = 0, 1, 2, 3



Compared to Repeated Measures ANOVA

- MLM and RM-ANOVA are the same in some basic situations
- Some advantages of MLM
  - Handles missing observations for individuals
    - Larger statistical power
  - Accommodates varying occasions
  - Allows clustering at a higher level (i.e., 3-level model)
  - Can include time varying or time-invariant predictor variables

Random Slope of Time

- It is uncommon to expect the growth trajectory is the same for every person
- Therefore, usually the <u>baseline model</u> in longitudinal data analysis is the <u>random coefficient model of time</u>

R Output (`glmmTMB`)

```
 Family: gaussian  ( identity )
Formula:          read ~ time + (time | id)

Conditional model:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  2.69609    0.04531   59.50   <2e-16 ***
time         1.11915    0.02183   51.27   <2e-16 ***
```

The estimated mean of read at time = 0 is γ_{00} = **2.70 (*SE* = 0.05)**

The model predicts that the constant growth rate per 1 unit increase in time (i.e., **2 years**) is γ_{10} **= 1.12 (*SE* = 0.02)** units in read

```
Random effects:

Conditional model:
 Groups   Name        Variance Std.Dev. Corr
 id       (Intercept) 0.57310  0.7570
          time        0.07459  0.2731   0.29
 Residual             0.34584  0.5881
```

What do the *SD*s mean?

Piecewise Growth

Alternative Growth Shape

- For many problems, a linear growth model is at best an approximation
- Other common models (need 3+ time points)
  - Piecewise
  - Polynomial
  - Exponential, spline, etc

Piecewise Growth Model

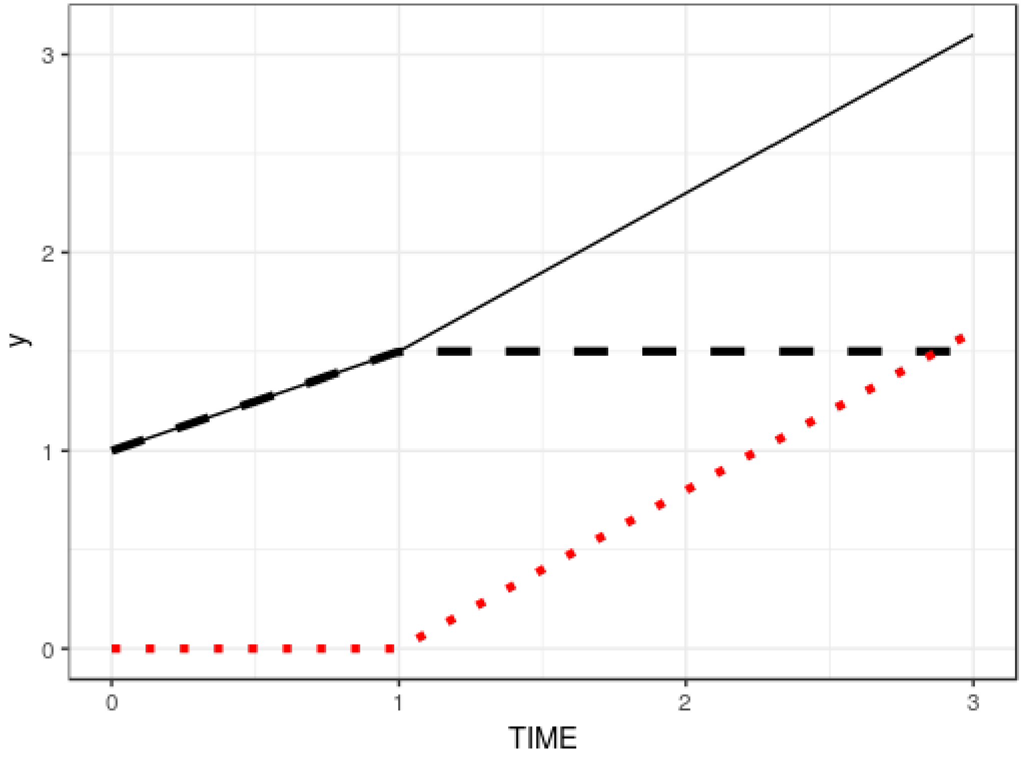
- Piecewise linear function
  - $Y = \beta_0 + \beta_1$ TIME, if TIME ≤ TIME^c
  - $Y = \beta_0 + \beta_1 \text{TIME}^c + \beta_2 (\text{TIME} - \text{TIME}^c)$, if TIME > TIME^c
- β_0 = initial status (when TIME = 0)
- β_1 = phase 1 growth rate (up until TIME^c)
- β_2 = phase 2 growth rate (after TIME^c)

Coding of Time

time	phase1	phase2
0	0	0
1	1	0
2	1	1
3	1	2

$b_0 = 1, b_0 = 0.5, b_2 = 0.8$

- Dashed line: Phase 1
- Dotted line: Phase 2
- Combined: Linear piecewise growth



R Output

```
Conditional model:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  2.52272    0.04599   54.85   <2e-16 ***
phase1       1.56213    0.04270   36.59   <2e-16 ***
phase2       0.87935    0.02548   34.52   <2e-16 ***
```

The model suggests that the average growth rate in phase 1 is 1.56 unit per unit time (*SE* = .04), but the growth rate decreases to 0.88 unit/time (*SE* = 0.03) subsequently.

R Output

```
Random effects:

Conditional model:
 Groups   Name        Variance Std.Dev. Corr
 id       (Intercept) 0.60520  0.7779
          phase1      0.22695  0.4764    0.13
          phase2      0.05364  0.2316   -0.15  0.96
 Residual             0.25144  0.5014
Number of obs: 1325, groups:  id, 405
```

SD of the phase 1 growth rate is 0.48. Plausible range: majority of children have growth rates between 1.56 +/- 0.48 = [1.08, 2.04]

SD of the phase 2 growth rate is 0.23. So majority of children have growth rates between 0.88 +/- 0.23 = [0.65, 1.11]

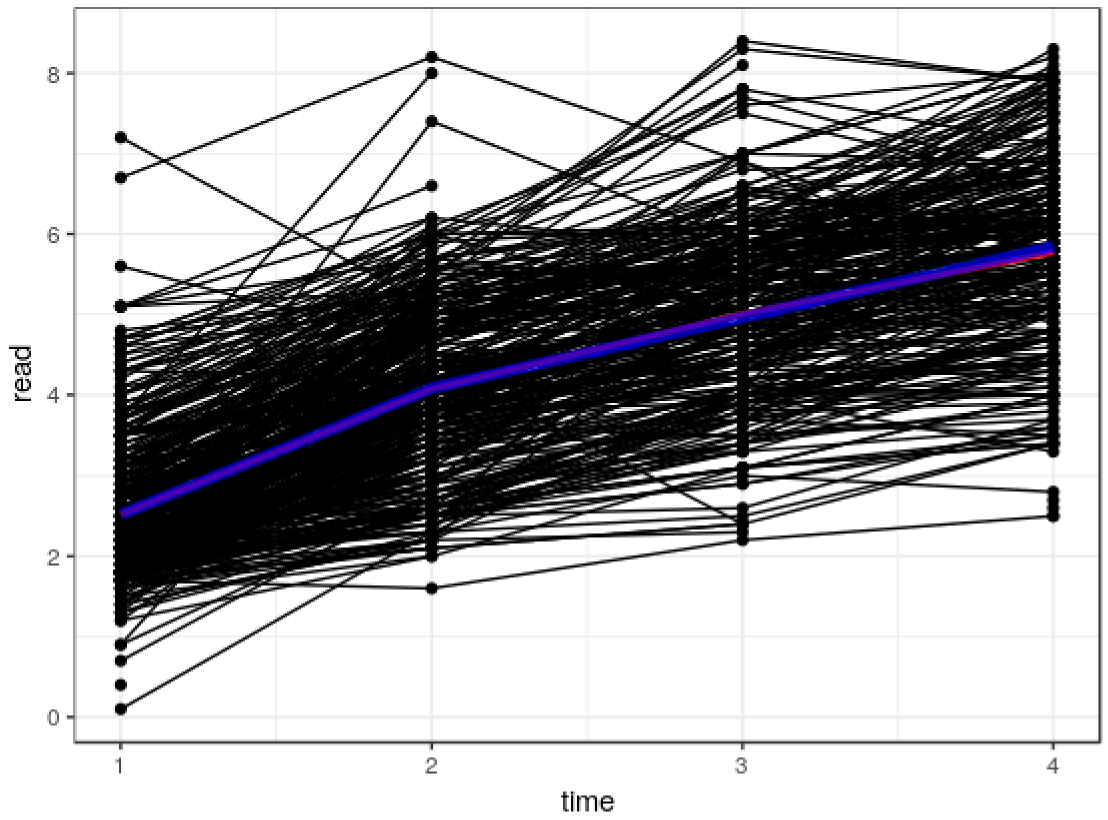
Model Comparison

```
> AIC(m_gca, m_pw)

       df      AIC
m_gca   6 3394.001
m_pw   10 3229.738
```

- The model with lower AIC should be preferred

Predicted Average Trajectory



Including Predictors

Time-Invariant vs Time-Varying Covariates

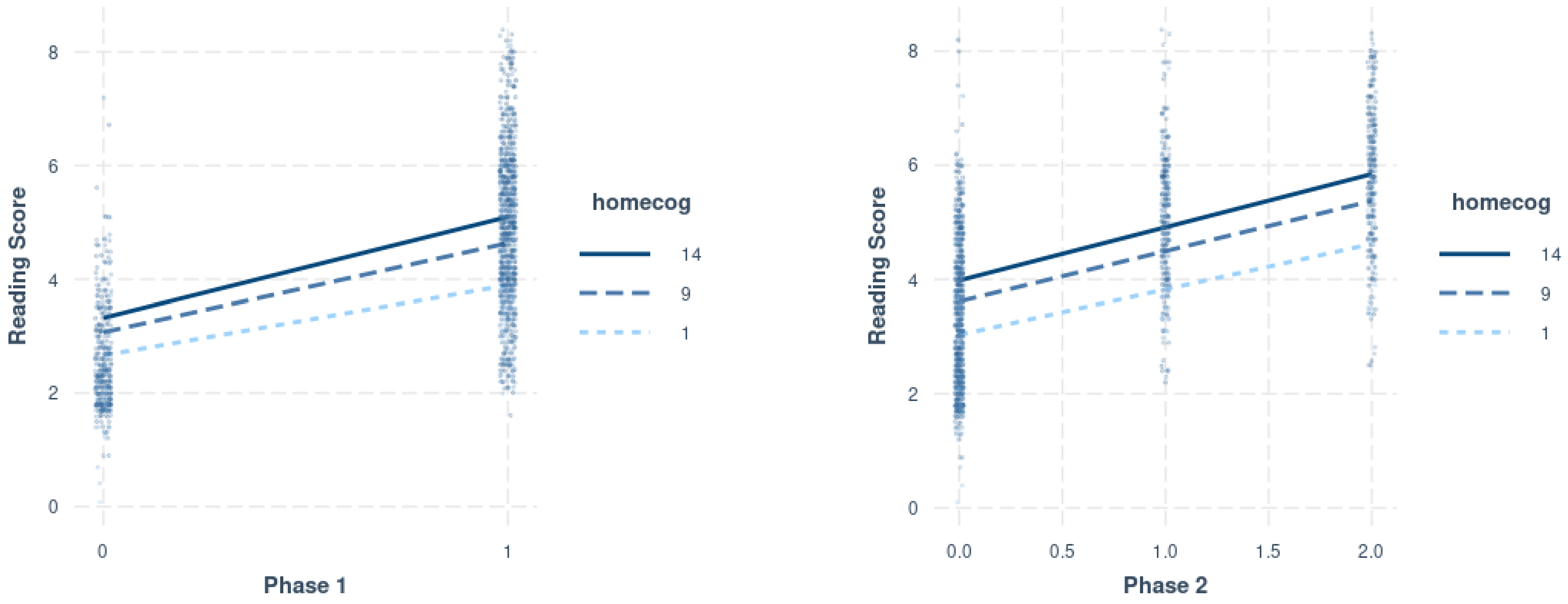
- Time-invariant predictor: Lv-2
- Time-varying predictor: Lv-1 (to be discussed next week)
  - "Cluster"-mean centering is generally recommended
  - However, usually not meaningful for "time." *Why?*

Time-Invariant Covariate

- Time-invariant predictor: Lv-2
  - Homecog (1-14): mother's cognitive stimulation at baseline
    - Centered at 9

```
Conditional model:
                Estimate Std. Error z value Pr(>|z|)    
(Intercept)      2.52735    0.04574   55.25   <2e-16 ***
phase1           1.56669    0.04240   36.95   <2e-16 ***
phase2           0.87980    0.02546   34.56   <2e-16 ***
homecog9         0.04364    0.01777    2.46   0.0140 *  
phase1:homecog9  0.04152    0.01661    2.50   0.0125 *  
phase2:homecog9  0.01051    0.01007    1.04   0.2964    
```

Cross-Level Interactions

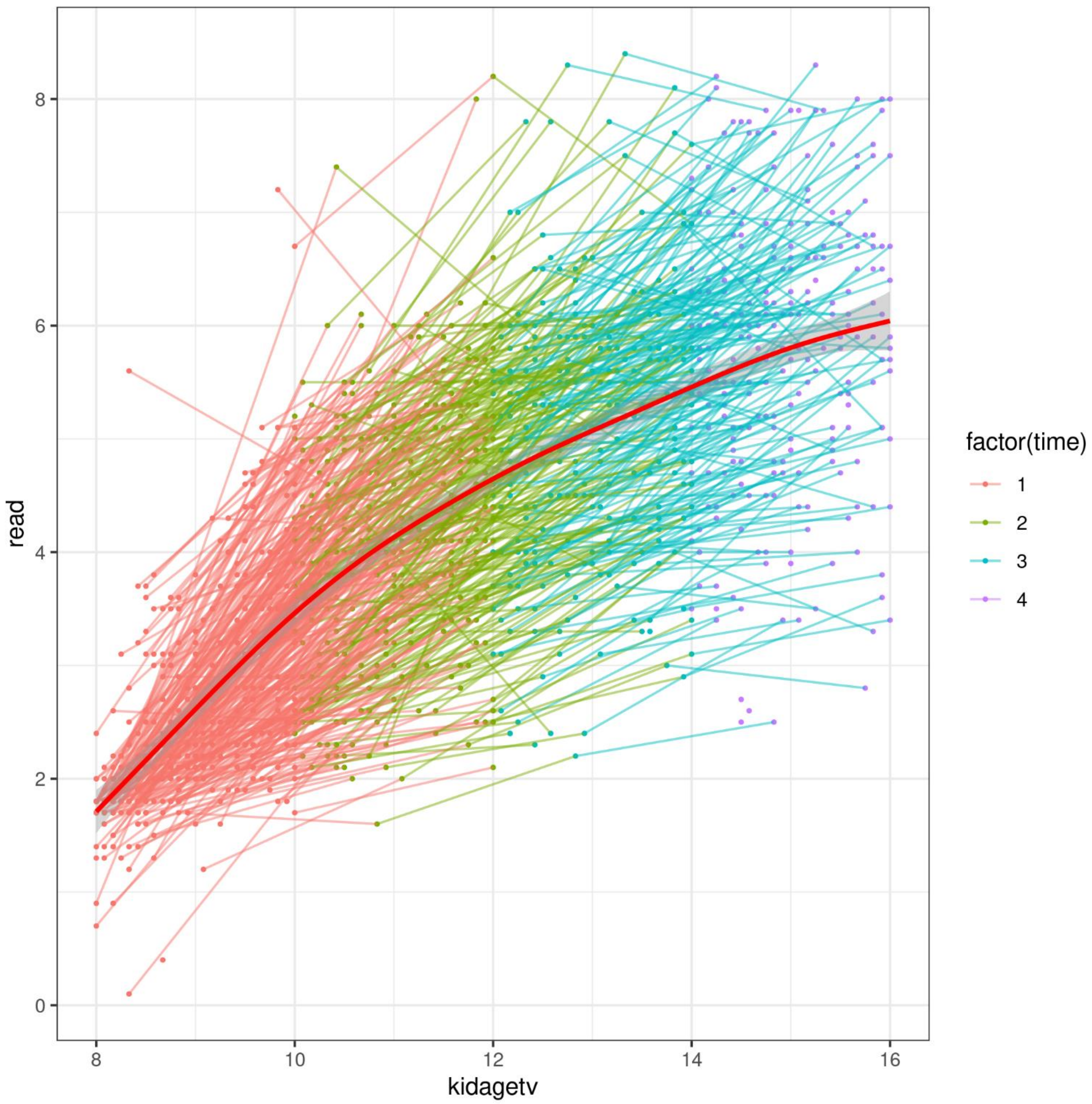


Handling Varying Occasions

Different “Time” Variables

- So far we model changes as a function of time passage from a common fixed point of history
  - I.e., when the study started
- In developmental research, one may be more interested in changes as a function of age
  - I.e., time passage from each person’s date of birth

- An advantage of MLM is that it does not require equal time intervals
  - Person 1: age 7 → age 9 → age 10
  - Person 2: age 5 → age 6.5 → age 8



read
kidagetv
factor(time)
1
2
3
4
0
2
4
6
8
8
10
12
14
16

Handling Varying Occasions

- Age as predictor (see textbook)

```
# Subtract age by 6
curran_long <- curran_long %>%
  mutate(kidagetv = kidage + time * 2,
         # Compute the age for each time point
         kidage6tv = kidagetv - 6)
# Fit the model
m_agesq <- glmmTMB(read ~ kidage6tv + I(kidage6tv^2) + (kidage6tv + I(kidage6tv^2) | id),
                   data = curran_long, REML = TRUE)
summary(m_agesq)
```

```
Random effects:

Conditional model:
 Groups   Name          Variance  Std.Dev. Corr
 id       (Intercept)   0.2941947 0.5424
          kidage6tv     0.1904793 0.4364   -0.98
          I(kidage6tv^2) 0.0009304 0.0305    0.90 -0.96
 Residual               0.2461501 0.4961
Number of obs: 1325, groups:  id, 405

Conditional model:
                Estimate Std. Error z value Pr(>|z|)
(Intercept)    -0.320744   0.096834  -3.312 0.000925 ***
kidage6tv       1.128463   0.042191  26.746  < 2e-16 ***
I(kidage6tv^2) -0.049581   0.003483 -14.236  < 2e-16 ***
```

The model suggests that the average initial growth rate is 1.13 unit per year (*SE* = .04)

The growth rate slows down by .05 every year. Therefore, at Wave 2 (two years later), the growth rate is 1.02

```
Random effects:

Conditional model:
 Groups   Name           Variance  Std.Dev. Corr
 id       (Intercept)    0.2941947 0.5424
          kidage6tv      0.1904793 0.4364   -0.98
          I(kidage6tv^2) 0.0009304 0.0305    0.90 -0.96
 Residual                0.2461501 0.4961
Number of obs: 1325, groups:  id, 405

Conditional model:
                Estimate Std. Error z value Pr(>|z|)
(Intercept)    -0.320744   0.096834  -3.312 0.000925 ***
kidage6tv       1.128463   0.042191  26.746  < 2e-16 ***
I(kidage6tv^2) -0.049581   0.003483 -14.236  < 2e-16 ***
```

The 68% plausible range of the initial growth rate is 1.13 +/- 0.44 = [0.69, 1.57]

The 68% plausible range of the change in growth rate is -0.05 +/- 0.03 = [-0.02, -0.08]